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APPLYING THE LAWS OF LOGIC TO THE LOGIC OF LAW

*Hillel Bavli**

INTRODUCTION

Consistency is a necessary condition of a just legal system, without which arbitrariness, unequal treatment, unpredictability, and, ultimately, injustice must result. “The truth,” remarked Justice Holmes, “is that the law is always approaching, and never reaching, consistency.”¹ But beyond meager intuition, or bare observation, is it possible to rigorously examine internal logical consistency²—mutual compatibility among legal deductions—in the rule of law?³

Kurt Gödel, in a 1931 publication of a German scientific periodical, disproved the then-common assumption that each area of mathematics can be sufficiently axiomatized as to enable the development of an “endless totality of true propositions” about a given area of inquiry.⁴ Specifically, he proved that any formal logical system (a concept that I shall more clearly explain below) that entails sufficient means as to support elementary arithmetic⁵ is necessarily subject to the inherent characteristic of incompleteness: arithmetical propositions which can be neither proved nor disproved within the system.⁶ Impliedly, every such system

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1. Mark R. Brown & Andrew C. Greenberg, *On Formally Undecidable Propositions of Law: Legal Indeterminacy and the Implications of Metamathematics*, 43 HASTINGS L.J. 1439, 1450 (1992) (quoting O. W. HOLMES, JR., *THE COMMON LAW* 36 (1881)).

2. Compare *external* consistency, which relates deductions to truth external to the relevant system. See DOUGLAS R. HOFSTADTER, *GÖDEL, ESCHER, BACH: AN ETERNAL GOLDEN BRAID* 94-95 (1999).

3. See ERNEST NAGEL & JAMES R. NEWMAN, *GÖDEL’S PROOF* 13 (Douglas R. Hofstadter ed., rev. ed. 2002); HOFSTADTER, *supra* note 2, at 94-100.

4. NAGEL & NEWMAN, *supra* note 3, at 4.

5. Arithmetic concerns the relations between whole numbers. KURT GÖDEL, *ON FORMALLY UNDECIDABLE PROPOSITIONS OF PRINCIPIA MATHEMATICA AND RELATED SYSTEMS I* (Dover ed. 1992).

6. *Id.* at 1, 57.

necessarily inheres either incompleteness or inconsistency. Further, Gödel proved the impossibility of *establishing* “internal logical consistency of a very large class of deductive systems . . . unless one adopts principles of reasoning so complex that their internal consistency is as open to doubt as that of the systems themselves.”⁷

If applicable to law (a significant contingency indeed), Gödel’s proof indicates unavoidable judicial susceptibility to inconsistency, since abstinence from adjudication of formally undecidable cases is impractical. Thus, application of Gödel’s Incompleteness Theorem to the legal context would establish *a priori* limitations on the capacity for consistency to exist within the law, as well as on the faculty to *establish* internal logical consistency within the law.

Perhaps more importantly, the law itself manifests plausible limitations on its capacity to realize formal consistency, or to be examined with respect to its consistency. Specifically, formalizing a logical axiomatic legal system—a requirement of rigorously examining internal logical consistency—that retains the fundamental values of justice may prove difficult if not impossible. Further, proving or disproving formal legal consistency may require construction of a legal language sufficiently exact to map, or mirror, meta-legal statements—statements *about* a formalized legal system—within the legal language itself. Such construction may prove impossible as well.

I begin by discussing the difficulties of proving consistency within a formal system generally. After establishing the importance of a formalized legal model as a prerequisite of rigorous examination of consistency, I investigate issues *intrinsic* to the current system of law that may prevent formalization of a just legal system as currently conceived.⁸ I argue that flexibility inherent in a just legal system (in the sense that judges have the ability to modify, in response to a given case, the presumptions from which that case’s outcome will be derived) may foreclose the possibility of legal formalization or any comprehensive model thereof. I conclude, however, that a model whose purpose is the examination of consistency within a system need not necessarily retain the dynamic nature of real-world formalization. Rather, a static model of legal formalization may avoid the complications confronting a comprehensive formalization of law, while retaining the fundamental values critical to examination of consistency within the law.

7. NAGEL & NEWMAN, *supra* note 3, at 5.

8. I abstain from expounding upon the common topic of “human intuition” in the law and its implications with regard to formalization.

I. PROVING CONSISTENCY: A SNAKE EATING ITS OWN TAIL

Suppose the creation of a system in which certain natural laws are presumed true. Further, specified rules are initially established to allow additional laws to be derived from the presumed natural laws, and to allow further additional laws to be derived from other derived laws, and so on. Let us appropriately call any law that is not a natural law a derived law.

Suppose that each year many new laws are derived from previously derived laws or from natural laws directly. Can it be shown that after many years beyond the system's creation, and many millions of laws beyond the initial natural laws, a contradiction among the system's laws will not arise? It is certainly invalid to conclude that a contradiction will not or cannot arise from the fact that one has not already arisen.⁹

It can be shown that internal consistency among the foundational natural laws necessarily implies consistency among further properly derived laws.¹⁰ Thus, to prove impossibility of contradiction among millions of eventual derived laws, one must prove consistency among the relatively few natural laws (assuming proper derivation). The possibility of such a proof—namely, that of consistency among the assumed foundational postulates of a given system—represents the concern of the current section.

A. The Axiomatic Method

Pure mathematics can be described as a science of deduction. It is the “subject in which we do not know what we are talking about, or whether what we are saying is true.”¹¹ Its concern is not the truth of the assumed postulates or the deduced conclusions, but only that its deductions follow as necessary logical consequences of its assumptions.¹² The “axiomatic method,” discovered by the ancient Greeks, is a system of deriving propositions, or theorems, from accepted postulates known as axioms.¹³ In the aforementioned example, natural laws are the system's axioms, and derived laws are the system's theorems.¹⁴ The ancient Greeks utilized the axiomatic method to develop an incredibly complex system of geometry deduced from five simple axioms (e.g., a straight line segment can be drawn joining any two points).¹⁵

9. NAGEL & NEWMAN, *supra* note 3, at 18-19.

10. *Id.* at 13.

11. *Id.* at 12 (citing “Russel’s famous epigram”).

12. *Id.* at 11.

13. *See id.* at 2.

14. *See supra* Part I (introductory paragraph).

15. HOFSTADTER, *supra* note 2, at 90. Euclid’s five axioms state:

The Euclidean axioms were presumed to be true statements about space. Thus, insofar as theorems were deduced from such axioms, the possibility of deducing a contradiction escaped consideration.¹⁶ That is, until the discovery of a new, non-Euclidian geometry.

The Nineteenth Century discovery of different, yet equally valid, systems of geometry, such as elliptical geometry, destroyed the crutch upon which faith in the consistency of Euclidian geometry rested: external truth.¹⁷ How can differing conceptions of a point or line both be true when only a single reality exists?¹⁸ The notion of mathematics as a real-world, rather than an abstract discipline was hereby challenged.¹⁹ Establishing the internal consistency of such systems suddenly took on critical importance.

B. Solving One Problem by Creating Another

The task of rigorously establishing the internal consistency of a system—even a simple system—quickly encounters a significant difficulty; specifically, a problematic set of alternative approaches. One approach is to utilize the system’s own rules and postulates to establish its consistency. It is difficult, however, to justify reliance on a given system of reasoning to prove consistency in that same system of reasoning. Such analysis is circular and therefore unfounded. “It is like lifting yourself up by your own bootstraps.”²⁰

The alternative to grounding a system’s own reasoning in itself is to establish grounding in a second system’s reasoning. This approach, however, accomplishes little more than shifting the problem to another domain.²¹ The proof of one system’s consistency becomes reliant upon the consistency of an additional system. A rigorous proof of the former system’s consistency would thus require proof of the latter system’s

(1) A straight line segment can be drawn joining any two points.

(2) Any straight line segment can be extended indefinitely in a straight line.

(3) Given any straight line segment, a circle can be drawn having the segment as radius and one end point as center.

(4) All right angles are congruent.

(5) If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. *Id.*

16. NAGEL & NEWMAN, *supra* note 3, at 13.

17. By “external truth,” I refer to truth external to the Euclidean geometric system.

18. *See* HOFSTADTER, *supra* note 2, at 19-21. Today, the possibility of multiple truths is apparent; then, however, it was not. *Id.* at 20.

19. *See id.* at 19.

20. *Id.* at 24.

21. NAGEL & NEWMAN, *supra* note 3, at 17.

consistency, which, in turn, must face the same difficulties plaguing the former system's proof.²²

Thus, a dilemma unfolds: proving consistency of a given logical system seemingly requires the illogical reasoning of internal circularity, or of "solving" one problem by creating an equivalent one.

C. Metamathematics

The solution to the consistency dilemma resides in a critical distinction between mathematics and metamathematics. The issue at hand, the provability of the internal consistency of a formal system (which, for now, I shall assume to be a mathematical system), is not one that belongs to that system in the sense that it speaks the language and to the domain of that system. Rather, it speaks *about* the system. Statements *about* a mathematical system belong to what German mathematician David Hilbert prescribed the term "metamathematics," a language *about* mathematics.²³

For example, the expressions " $2 + 5 = 7$," " $X + 6 = 14$," and " $0 = 0$ " belong to the language of mathematics, while the expressions "7 is a prime number," "every number has a successor number," and "formal system X is consistent" belong to the language of metamathematics.²⁴

Mathematicians Russel and Whitehead produced *Principia Mathematica*, which set forth to create a universal and unambiguous language of mathematical reasoning.²⁵ *Principia Mathematica* purported to deduce all of mathematics from logic without contradiction.²⁶ It derived the axioms of number theory from formal logic, thus reducing the issue of the system's consistency to the question of consistency among the system of formal logic itself.²⁷ It was uncertain, however, whether the system contained all of mathematics, or whether the system of reasoning was even internally consistent. Thus, the issue of proving consistency once again prevails: could it be shown that no contradiction would ever be derived from the methods delineated in *Principia Mathematica*?²⁸ Hilbert challenged the world of mathematics to do just that!

Hilbert's program endeavored to skirt the aforementioned difficulties resulting from relative proofs of consistency by constructing "absolute" proofs, whereby proof of a system's consistency would not require the

22. *See id.* at 7-24.

23. *See id.* at 27-28; GÖDEL, *supra* note 5, at 1-2.

24. *See* NAGEL & NEWMAN, *supra* note 3, at 25-36.

25. *See* HOFSTADTER, *supra* note 2, at 23.

26. *See id.*

27. NAGEL & NEWMAN, *supra* note 3, at 43.

28. *See* HOFSTADTER, *supra* note 2, at 23.

assumption of consistency in a second system. Hilbert sought a proof of consistency or completeness that relied only upon “finitistic” models of reasoning (a concept beyond the scope of the current paper).²⁹ His program was soon to be shattered by Gödel’s proof.³⁰

II. GÖDEL’S INCOMPLETENESS THEOREM

Kurt Gödel revolutionized the study of systems and the field of mathematics in his 1931 publication of the paper *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*.³¹ Gödel addressed the difficulties of proving consistency in formal systems by ingeniously employing a mathematical tool known as “mapping.” He thereby laid to rest the ancient question of completeness and consistency in formal mathematics. His conclusions, however, were not those sought or expected by the mathematics community.³² Gödel’s proof shattered the possibility of complete and consistent axiomatization. In so doing, he sent shockwaves through the worlds of mathematics, the sciences, and philosophy.

A. Mapping

Mapping is a fundamental mathematical technique used to mirror concepts from one domain upon another while retaining their abstract structure and relation. For example, geometric terms are mirrored in algebra, spherical shapes are projected on geometric planes, and electric currents are mapped in hydrodynamics.³³ Gödel utilized this feature of mapping to translate complicated metamathematical statements about a formalized system of arithmetic into arithmetical statements within the system itself.³⁴ Gödel’s implementation of mapping enabled him to transcend the difficulties and limitations of proving consistency in the language of metamathematics (specifically, to transcend the complexities of proof through finitistic metamathematical models).³⁵

B. Incompleteness or Inconsistency

Proposition VI of Gödel’s proof asserts that any formal system

29. *See id.*

30. *See id.*

31. *See id.* This paper was later published under the same title. *See supra* note 5.

32. *See supra* notes 25-29.

33. *See* NAGEL & NEWMAN, *supra* note 3, at 64.

34. *Id.*

35. *See id.* at 66.

sufficiently complex to support arithmetic³⁶ must contain statements that are either internally “undecidable”—statements that cannot be proved or disproved within the system—or provably inconsistent.³⁷ Proposition VI proves that every such formal system must be incomplete or inconsistent.

Gödel constructed a formula, G , within arithmetic, representing the statement, “The formula G is not provable within the system.”³⁸ He demonstrated that G is provable if, and only if, its negation, $\sim G$, is provable.³⁹ (Try it! Ask yourself whether G has a proof within the system.) If both a formula and its negation are provable within a system, then the system is, by definition, inconsistent. Alternatively, if the system is consistent, then both G and $\sim G$ are not provable within the system.⁴⁰ Gödel’s proof resulted in the recognition that performance of higher mathematics requires some degree of “informal metamathematical reasoning.”⁴¹

III. LIMITATIONS ON FORMAL EXAMINATION OF CONSISTENCY WITHIN THE LAW

The key ingredient to rigorous examination of the logical consistency of a system is the formalization of that system.⁴² Up to this point, I have referred to “formal systems” and “formalization” with relative informality. I now explain exactly what is meant by such terms.

A. Formalization

A formal system is a process of reasoning by which “truths” are deduced from the system’s assumptions, or axioms. A formal system consists of three ingredients: 1) a definite (objective) language of symbols and syntax; 2) a set of axioms defined as, “a finite list of general propositions whose truth, given the meanings of the symbols, is supposed to be self-evident”; and 3) a set of rules by which new propositions may be inferred from axioms and established propositions.⁴³

36. Practically speaking, a system need not be very complex to support arithmetic.

37. GÖDEL, *supra* note 5, at 62.

38. *See id.* at 63.

39. *Id.*

40. *See id.* at 57-62. *See also* NAGEL & NEWMAN, *supra* note 3, at 92-93; HOFSTADTER, *supra* note 2, at 17-19.

41. *See* Brown & Greenberg, *supra* note 1, at 1468.

42. *See* GÖDEL, *supra* note 5, at 4; NAGEL & NEWMAN, *supra* note 3, at 10-14.

43. Brown & Greenberg, *supra* note 1, at 1445 (quoting ROGER PENROSE, THE EMPEROR’S NEW MIND: CONCERNING COMPUTERS, MINDS, AND THE LAWS OF PHYSICS 102-05 (1989)).

“The system’s rules must be defined solely in the system’s language,” and must be solely in relation to “other rules within the system.”⁴⁴ The fundamental component of a formal system is that its derivations depend solely upon the system’s rules of inference and its initial axioms.⁴⁵ Deductions within a formal system are definite and conclusive.

Note that a formal system does not require that *any* question be determinately deducible, only that every deduction be complete, consistent, and resultant solely from the axioms and rules of inference. A designedly formal system that is unable to answer certain problems while maintaining consistency is said to be “indeterminate” (Gödel’s theorem thus places limits on the determinacy of formal systems).⁴⁶

I use the term “formalization” to refer to the process of transforming an informal system into a formal one while retaining the system’s basic properties. Thus, formalization of an informal system of law refers to creating an objective legal language, a set of axioms, and definite rules of inference, such that all results are conclusive deductions solely from the system’s axioms and rules of inference, while retaining the system’s basic properties.

Consider the importance of formalization with regard to examination of a system’s internal consistency.⁴⁷ The reliance on “truth” external to a system was a flaw that crushed ancient systems of reasoning.⁴⁸ As mentioned above, pure mathematics—the science of deduction—has no concern for “truth,” but only that its deductions are necessary and logical consequences of its axioms.⁴⁹

Entertain the question of consistency in a system that lacks an objective language, a definite set of axioms, and rules of inference—a system that lacks formalization. The concept of consistency within such a system is difficult to imagine, if not utterly nonexistent. Consider proving or disproving consistency between one “derivation” and another in a system lacking definite axioms from which “derivations” are to be deduced, or rules by which “derivations” are to be inferred, or a language in which “derivations” are to be expressed with objectivity. The three components of the formal system are critical to the examination of consistency in the current understanding of the concept.

44. *Id.*

45. *See id.*

46. *See id.* at 1444-48.

47. It may be helpful to review the section on the axiomatic method, *supra* notes 9-19 and accompanying text.

48. *See supra* notes 16-19 and accompanying text.

49. *See supra* notes 11-19.

IV. MODELING LAW AS A FORMALIZED SYSTEM

The law in its current state is not formalized. It is true, however, that only rarely do transactions become controversies, and seldom do controversies become court cases, indicating that the legal system does, in fact, enable parties to apply legal deductions accurately in the vast majority of cases. (Consider the multitude of transactions occurring each day of your life. How many of them result in court cases?). Additionally, most court cases are easily determinable and even those that are not usually result in agreement among judges and scholars.⁵⁰ Only rarely do divided opinions and interpretations result, and such divergence may result from discrepancies in levels of intelligence or judicial error.⁵¹

The law, however, does allow for a great diversity of opinion and interpretation. Legal language is vague; statutes and rulings are wide open to interpretation; various conceptions of law—all valid—create discrepancies in judicial focus and notions of justice; and “justice” itself often promotes indeterminacy. However, of relevance is not whether law is or is not formal, but whether law can or cannot be modeled as a formal system for purposes of examination of consistency within the law.⁵²

A. Legal Language

Legal English is the language of law in the United States. At issue is whether legal English (or any language of equivalent complexity) is sufficiently exact as to satisfy the first requirement of a formal system—a definite and objective language of symbols and syntax. Arguably, language is, by its very nature, subjective, thus affording the judiciary free reign to interpret the law (such as statutes and precedent) and decide cases in accordance with its own standards and intentions.⁵³ Such utter subjectivity, however, likely overstates the case. More plausible is the view that *aspects* of language are vague, but that generally language is quite clear. Further, it is doubtful that language can appropriately be dichotomized between clear and unclear concepts. Rather, varying degrees of clarity likely exist within a continuum. Thus, the issue turns upon the

50. Supreme Court cases, however, are particularly selected for their difficulty, thus resulting in divergent opinions and interpretations. See Frederick Schauer, *Easy Cases*, 58 S. CAL. L. REV. 399, 409 (1985); see also Brown & Greenberg, *supra* note 1, at 1452.

51. Brown & Greenberg, *supra* note 1, at 1452-54.

52. Note that a discussion of the advantages and disadvantages of formalization is very interesting but exceeds the scope of the current paper, which is concerned with formalization insofar as it allows examination of consistency within the legal system.

53. See Stanley Fish, *Interpretation and the Pluralist Vision*, 60 TEX. L. REV. 495, 503 (1982); Stanley Fish, *Wrong Again*, 62 TEX. L. REV. 299, 314 (1983).

degree of precision necessary to establish objectivity. Since formal systems remain primarily a concept belonging to mathematics, let us assume sufficient (but not necessarily required) objectivity in languages of equal or greater clarity than that of mathematics.⁵⁴

It is quite clear that legal language has yet to achieve a level of definition even close to that achieved by mathematics. Recall, however, that the question at hand is not whether legal language *does* satisfy the demands of formalism, but rather whether it *can* satisfy such demands. It may be that relative objectivity may only exist at the sacrifice of another property that is fundamental to a just legal system. Perhaps subjectivity is a virtue in itself! Whatever the case may be, until valid proof is offered one way or the other, which is not the case currently, the possibility of objectivity in the language of law cannot be ignored. Further, while the complexities of legal language may limit the practicability of producing an objective language, its accomplishment in mathematics evinces otherwise (although, arguably, mathematics does not entail at least equivalent complexity). Suffice it to say that impracticability must not be assumed.

B. Implanting the Heart and Circulation of a Formal System: Its Axioms and Rules of Inference

The heart of a formal system is its set of axioms and rules of inference. This section examines the potential for establishing rules of legal reasoning and axiomatizing the law while retaining its fundamental values.

It is easy to imagine a determinate formal system of law that fails to retain its fundamental values. For example, a legal system consisting of the rule, “All plaintiffs lose,” provides a complete, consistent, and conclusive ruling for each case.⁵⁵ However, the system fails to retain the current system’s notion of justice (or anything closely related to it) and is therefore uninteresting for purposes of examining consistency in the law. Thus, it is of fundamental importance to design a formal model only while preserving the object system’s values.

Rather than discuss the heavily-debated role of “intuition” in the law, and its implications with regard to the possibility of legal formalization, my goal in the following sections is to expound upon issues intrinsic to a just legal system which may preclude the possibility of modeling a formalized system of law. Specifically, I discuss problems inherent in formulating a single and definite set of axioms, as well as in axiomatizing law generally.

54. See Brown & Greenberg, *supra* note 1, at 1458-59.

55. See Joseph W. Singer, *The Player and the Cards: Nihilism and Legal Theory*, 94 YALE L.J. 1, 5-6 (1984); see also Brown & Greenberg, *supra* note 1, at 1463.

A discussion of the various functions of the legal system is well beyond the scope of the current paper. Thus, I assume that justice is the primary goal of the legal system. My analysis, however, requires only that justice be a fundamental value in the system.

1. Modeling a Single Set of Axioms from a Multitude of Such Sets

Imagine a formalized legal system whereby all statutes and common-law precedent form the system's axioms (assume that statutes and precedent embody societal values like fairness and efficiency). New rules in the form of judicial rulings are produced only by applying the system's axioms to a set of facts (presumed to be known) and deducing logical inferences from established axioms to produce rules explicitly directed to such facts.

The model's oversimplification reveals many problems that would similarly apply to more complex models. One such problem is the requirement that a formal system feature a finite list of propositions, the truth of which is presumed—a definite set of axioms. In the simplified example provided, a definite set of axioms composed of statutes and precedent becomes a multitude of sets when considered with respect to each judge (and other administrators of the law), since no two judges maintain identical interpretations of the law. Each judge's interpretation translates to its own set of axioms—and that assumes that each judge has a clear idea of his interpretation in the first place.

A model may arguably circumvent the problem of multitudinous sets of axioms by assuming the potential of amalgamating contradictory sets to produce a single set of axioms. Similarly, a model may approach the law from the perspective of a single judge.

Thus, the issue turns upon the propriety of such models. At least two objections can be made to them: first, that models assuming amalgamation of many subjective interpretations, or law as conceived by a single judge, ignore real-world conditions, and are therefore trivial in value; second, that such models fail to align with common conceptions of justice within the legal system.

Formulation of a single set of axioms that encompasses the multitude of meanings—even contradictory meanings—attached to the law is doubtful, if not, by definition, utterly impossible. Perhaps more feasible is the establishment of a set of axioms that *indirectly* embodies innumerable interpretations of the law by achieving consensus on each axiom. The law is, of course, multifaceted such that a simple average could never apply. However, just as statutes are established in the first place through negotiation, compromise, and ultimately consensus, a second order consensus as to the interpretation of statutes and precedent may be

achievable. Of course, interpretations may differ on interpretations, thus requiring a third order consensus, and so on and so forth, until, eventually, the importance of consensus on higher order interpretations is phased out. However, for the sake of simplicity, I will ignore this complication.

Assuming the possibility of modeling a single set of axioms embodying the law's multitudinous interpretations, doing so may chip away properties fundamental to a just legal system, and, thus, render the model barren. Specifically, the concept of competing axioms may be critical to common notions of justice in the law. Of course, if the virtue of competing axioms is moderation via compromise, the same may be achieved through the aforementioned second-order consensus.

If, on the other hand, the value is *intrinsic* to multiple sets of competing axioms, the property is irreplaceable. Such value, however, is not easily found. In fact, plausibility lies in the contrary: competing axioms are likely detrimental insofar as they generate arbitrariness and unequal treatment in the law. A case may have one result before one judge and a contrary result before a second judge. But perhaps a degree of comfort is found in small doses of arbitrariness, better known as luck. The question of intrinsic value attached to competing axioms is likely an empirical one.

Further, the question at hand concerns consistency in the legal system generally, not consistency in the administration of law by any single judge. The relevance of a single-judge model is therefore questionable, since the model would embody the values of an individual judge, rather than those of the judiciary as a whole. However, a single-judge model may, nevertheless, provide a stepping stone to creating a more realistic model; namely, one that characterizes correspondence among contradictory sets of axioms respectively applied by a multitude of judges.

2. *Flexibility: A Concept Diametrically Opposed to Axiomatization*

A formal system features a definite set of axioms solely from which conclusions within the system (e.g., theorems or rules) are deduced. A formal legal model would, therefore, require that rulings be derived exclusively from its established axioms. Herein lies a second, perhaps more serious, issue with modeling law as a formal system. Specifically, it may be impossible to maintain such requirements while retaining an element that is perhaps fundamental to justice: flexibility.

"Flexibility," as I use the term, refers to a system's capacity to modify a set of axioms—to use the jargon of formalism—in light of the facts of the object case, or, the case to which such axioms are to apply. In other words, critical to the current conception of a just legal system is the system's ability to "overturn," or "reverse," previously created precedent (axioms),

or even simply to produce new law (to add new axioms), rather than applying precedent (drawing deductions solely from the current set of axioms). The issue of such flexibility may pose a fatal blow to the possibility of establishing a formal system of law, or, likewise, modeling such a system.

Flexibility, as I use the term, is a concept diametrically opposed to that of axiomatization. In fact, it may be oxymoronic even to define “flexibility” as an ability to modify the set of axioms in light of the facts of the object case, since where such flexibility exists, a set of *definite axioms* cannot. Flexibility is undoubtedly a critical component of justice. Thus, the issue with regard to formalization is seemingly incurable.

It is essential to distinguish the current usage of “flexibility” from the term in its more common usage—namely, the law’s ability to account for “human” factors such as fairness or clemency. I assume that such “human” elements can, in fact, be incorporated within the system. For example, a given axiom may grant leniency, notwithstanding the system’s more substantive-based axioms, under certain predetermined circumstances, such as leniency for a mother whose sole motivation for stealing food from a supermarket was to provide food for her children. Thus, “flexibility” in the sense of allowing for certain variables other than strict application of substantive law—whether in the form of statutes or precedent, or of axioms other than those created by statutes or precedent—is not an issue effecting problems of formalization. Such flexibility may indeed be consistent with supplying a definite set of axioms.

The problem at hand is different. A judge’s ability to modify precedent, for example, is critical to the current system’s notion of justice. However, the ability to modify the axioms of a system in response to the object (case) to be determined conclusively by such axioms (and rules of inference) defeats the very nature of a formal system. A property intrinsic to justice is in direct contravention with the fundamentals of a formal system, thus inhering ineluctable incompatibility with formalization or any comprehensive model thereof. That is to say, a model of law as a formal system cannot be established insofar as it mirrors the dynamic nature of formalization just described. A model whose purpose is the examination of consistency within a system, however, need not necessarily retain the dynamic nature of real-world formalization.

V. A STATIC MODEL OF LAW AS A FORMAL SYSTEM

The problem described above is unavoidable with regard to formalization of the law. However, a *static model* of the law as a formalized system may manage to circumvent the flexibility problem

confronted by formalization itself.

Flexibility to modify the system's axioms in response to the case to be determined by such axioms renders each and every case, by definition, indeterminate. It further erodes the fundamental nature of a formal system: it liquefies the definitiveness of the system's axioms.

A model whose purpose is the examination of consistency may, however, circumvent the flexibility problem by maintaining a static character. While a dynamic model may, for example, relate currently established axioms to future cases (even cases in the immediate future), a static model details a system's axioms only insofar as they exist at any given time. In other words, a model that is dynamic in nature entertains future deduction, while a model that is static in nature recognizes a system's axioms and deductions only as the axioms currently exist. Thus, a static model may be of insignificant worth with respect to determination of case-outcomes (since flexibility allows modification of axioms). It may, however, be invaluable in examining a system's consistency, since such examination may *require* formalization. Unlike inquiry requiring a dynamic model, examination of the law's consistency at any given time may, in certain circumstances, require only a model with respect to the system's conditions, such as axioms, rules of inference, and *possible* deductions, as they exist at any such time: a static model.

CONCLUSION

The importance of examining consistency within the legal system cannot be overstated. Consistency is the ground upon which fundamental values of fairness are rooted. Rigorous examination of consistency requires contemplation of the law modeled as a formal system. Comprehensive modeling of legal formalization may be impracticable, just as formalization of law itself may be. However, simplified models of formalized systems of law may provide the fundamental elements necessary to examine consistency within the law while retaining the fundamental values of the system.

I have shown that formalization of law ineluctably requires the sacrifice of values critical to a just legal system, and that models of such formalization may, therefore, entirely spurious. However, a static model of formalization may circumvent certain problems extant in dynamic models while retaining the values that may allow examination of consistency within the law.